IOWA
End-of-Course
Assessment
Programs
Released Items
Let \( A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \) and \( B = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} \). What is the solution to the matrix equation \( AX = B \)?

A \( X = \begin{bmatrix} -4 & 3 \\ -2 & 1 \end{bmatrix} \)

INCORRECT: The student found
\[
A^{-1} = \frac{1}{3-4} \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = -1 \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix},
\]
then multiplied \( A^{-1} \) and \( B \) in the wrong order.
\[
X = BA^{-1}
\]

B \( X = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix} \)

INCORRECT: The student found the wrong \( A^{-1} \) as
\[
A^{-1} = 1 \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix},
\]
then multiplied \( A^{-1} \) and \( B \) in the wrong order.
\[
X = BA^{-1}
\]

C \( X = \begin{bmatrix} -6 & -5 \\ 4 & 3 \end{bmatrix} \)

CORRECT: The student found
\[
A^{-1} = \frac{1}{3-4} \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = -1 \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix},
\]
then multiplied \( A^{-1} \) and \( B \) in the correct order.
\[
X = A^{-1}B
\]

D \( X = \begin{bmatrix} 6 & 5 \\ -4 & -3 \end{bmatrix} \)

INCORRECT: The student found the wrong \( A^{-1} \) as
\[
A^{-1} = 1 \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix},
\]
then multiplied \( A^{-1} \) by \( B \).
\[
X = A^{-1}B
\]

CCSS Conceptual Category:
Algebra

CCSS Domain:
Reasoning with Equations and Inequalities
2 If matrix $A$ has dimensions of $4 \times 6$ and matrix $B$ has dimensions of $6 \times 2$, what are the dimensions of matrix $AB$?

A $2 \times 4$

INCORRECT: The student thought the dimensions of $AB$ were the number of columns of $B$ by the number of rows of $A$.

B $4 \times 2$

CORRECT: The student identified the dimensions of $AB$ as the number of rows in $A$ by the number of columns in $B$.

C $6 \times 6$

INCORRECT: The student thought the result should be a square matrix with the common dimensions of $A$ and $B$.

D $24 \times 12$

INCORRECT: The student thought the dimensions were the products of the row and the columns: $(4)(6) \times (6)(2)$.

**CCSS Conceptual Category:**
Algebra

**CCSS Domain:**
Reasoning with Equations and Inequalities
A volleyball tournament was played over a two-day period. Ticket prices were $6 for adults and $4 for students. Matrix $T$ shows the numbers of tickets sold for the tournament and matrix $C$ shows the ticket prices in dollars.

$$T = \begin{bmatrix} \text{Adult} & \text{Student} \\ \text{Day 1} & 200 & 325 \\ \text{Day 2} & 250 & 400 \end{bmatrix}, \quad C = \begin{bmatrix} \text{Adult} \\ \text{Student} \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

Which matrix operation gives the total revenue, in dollars, from ticket sales for each day of the tournament?

A $C \times T$

INCORRECT: The student did not take into account the dimensions of each matrix when determining the order of the product.

B $T \times C$

CORRECT: The student multiplied the number of tickets sold (2 × 2 matrix) by the ticket price (2 × 1 matrix).

C $2T \times C$

INCORRECT: The student tried to account for the 2-day event by multiplying by 2.

D $2(C \times T)$

INCORRECT: The student used the product (ticket price) × (number of tickets sold), and then tried to account for the 2-day event by multiplying that product by 2.

CCSS Conceptual Category:
Algebra

CCSS Domain:
Creating Equations
What should \[
\begin{bmatrix}
7 \\
4
\end{bmatrix}
\] be multiplied by to solve the matrix equation below?

\[
\begin{bmatrix}
2 & -2 \\
1 & 3
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
7 \\
4
\end{bmatrix}
\]

A \[\frac{1}{4} \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}\]

INCORRECT: The student found the determinant by adding, not subtracting, \(\frac{1}{(2)(3) + (-2)(1)}\) and interchanged the non-leading diagonal elements.

B \[\frac{1}{4} \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}\]

INCORRECT: The student found the determinant by adding, not subtracting, \(\frac{1}{(2)(3) + (-2)(1)}\).

C \[\frac{1}{8} \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}\]

INCORRECT: The student interchanged the non-leading diagonal elements.

D \[\frac{1}{8} \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}\]

CORRECT: The student found the determinant \(\frac{1}{(2)(3) - (-2)(1)}\), interchanged the leading diagonal elements, and changed the signs of the non-leading diagonals.

CCSS Conceptual Category:
Algebra

CCSS Domain:
Reasoning with Equations and Inequalities
What should \[
\begin{bmatrix}
2 & 0 \\
0 & 1
\end{bmatrix}
\] be multiplied by to obtain the matrix \[
\begin{bmatrix}
2 & 0 \\
0 & 1
\end{bmatrix}
\]?

A \[
\begin{bmatrix}
0.5 & 0 \\
0 & 1
\end{bmatrix}
\]
INCORRECT: The student found the inverse of the matrix.

B \[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]
CORRECT: The student used the identity matrix.

C \[
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]
INCORRECT: The student placed the 1's on the wrong diagonal.

D \[
\begin{bmatrix}
1 & 1 \\
1 & 1
\end{bmatrix}
\]
INCORRECT: The student thought the identity matrix has all elements of 1.

CCSS Conceptual Category: Algebra

CCSS Domain: Arithmetic with Polynomials and Rational Expressions