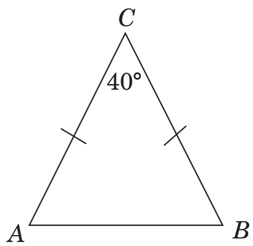
IOWA End-of-Course Assessment Programs

Released Items

GEOMETRY

1 **Given isosceles $\triangle ABC$ shown below, what is the measure of $\angle A$?**



A 40°

> INCORRECT: The student thought that $\angle A$ and $\angle C$ were the base angles and thus had the same measure.

B 50°

> INCORRECT: The student subtracted 40° from 90°.

C 60°

> INCORRECT: The student subtracted 40° from 100°.

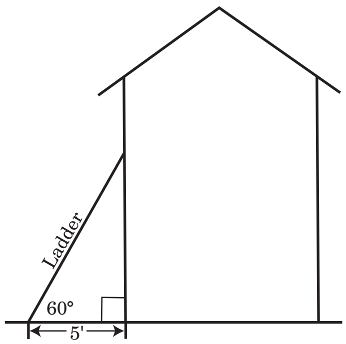
D 70°

> CORRECT: The sum of all interior angles in a triangle is 180°. $180° - 40° = 140°$. Since the triangle is isosceles the measures of the base angles ($\angle A$ and $\angle B$) will be equal: $\frac{140°}{2} = 70°$.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Congruence

2 **A ladder is placed against a house so that the base is 5 feet from the house and forms a 60° angle with the ground. Which expression represents the length of the ladder (in feet)?**



A $\dfrac{5}{\cos 60°}$

CORRECT: Cosine is the ratio of the adjacent leg to the hypotenuse in a right triangle. If x is the length of the ladder in feet, $\cos 60° = \frac{5}{x} \rightarrow x = \frac{5}{\cos 60°}$.

B $\dfrac{5}{\sin 60°}$

INCORRECT: The student used sine.

C $5(\cos 60°)$

INCORRECT: The student wrote the cosine ratio incorrectly as the hypotenuse to the adjacent leg.

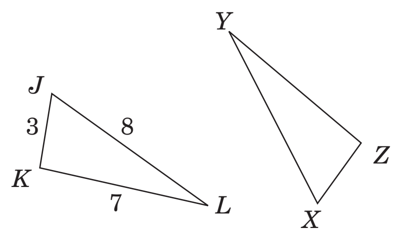
D $5(\sin 60°)$

INCORRECT: The student used sine and incorrectly wrote the ratio as the hypotenuse to the adjacent leg.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Similarity, Right Triangles, and Trigonometry

3 **Given that $\triangle JKL \cong \triangle XZY$, what is the length of $\overline{YZ}$?**



A 3

INCORRECT: The student thought $\overline{YZ}$ corresponds with $\overline{JK}$.

B 7

CORRECT: Since $\triangle JKL \cong \triangle XZY$ the pairs of corresponding sides are congruent. Side $\overline{YZ}$ corresponds to side $\overline{LK}$ so the length of $\overline{YZ}$ is 7.

C 8

INCORRECT: The student thought $\overline{YZ}$ corresponds with $\overline{JL}$.

D Cannot be determined

INCORRECT: The student misread the congruent symbol as a similarity symbol.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Congruence

4 **If the area of a square is 25 cm^2, what is the perimeter of the square?**

A 10 cm

INCORRECT: The student found 2 times the side length.

B 20 cm

CORRECT: Since the area of the square is 25 cm^2 then each side length (s) is $\sqrt{25}$ cm or 5 cm. The perimeter of a square is $4s$: $4(5) = 20$ cm.

C 25 cm

INCORRECT: The student thought the area and perimeter of a square are always the same.

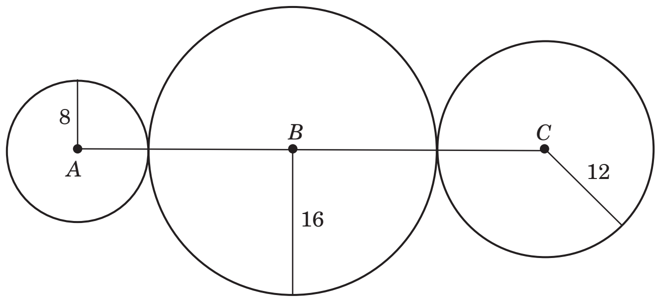
D 50 cm

INCORRECT: The student found the perimeter of the square by doubling the area.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Geometric Measurement and Dimension

5 **Given circles A, B, and C, what is the length of $\overline{AC}$?**



A 36

INCORRECT: The student added the radii of the circles.

B 48

INCORRECT: The student multiplied the radius of circle B by 3.

C 52

CORRECT: The length of $\overline{AC}$ is found by determining the sum of the radius from circle A, the diameter from circle B, and the radius from circle C. $8 + 32 + 12 = 52$.

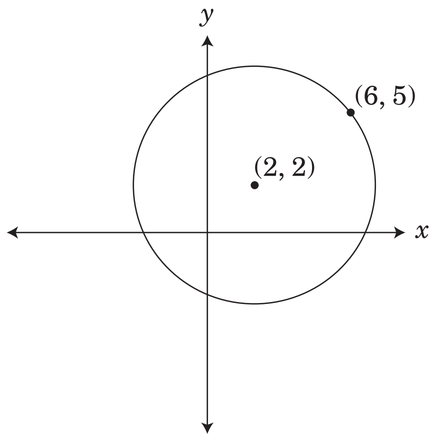
D 72

INCORRECT: The student added the diameters of the circles.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Circles

6 **The circle shown below has its center at (2, 2) and contains (6, 5). What is the length of the diameter of the circle?**



A 5 units

INCORRECT: The student found the radius of the circle.

B 7 units

INCORRECT: The student found the distance from the point (2, 2) to the point (6, 5) as $\frac{(6-2)+(5-2)}{2} = 3.5$ and then multiplied by 2.

C 10 units

CORRECT: Since the point (2, 2) is the center and the point (6, 5) is a point on the circle, the distance between these two points will determine the radius of the circle. Using the Distance Formula:

$D = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$D = \sqrt{(6-2)^2 + (5-2)^2}$

$D = \sqrt{(4)^2 + (3)^2}$

$D = \sqrt{16 + 9}$

$D = \sqrt{25}$

$D = 5$

Since the diameter (d) of the circle is twice the radius of the circle, $d = 2(5) = 10$ units

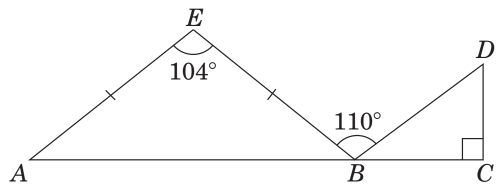
D 14 units

INCORRECT: The student found the distance from the point (2, 2) to the point (6, 5) as $(6-2) + (5-2) = 7$ and then multiplied by 2.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Expressing Geometric Properties with Equations

7 **In the figure below, $\overline{AE}$ is congruent to $\overline{BE}$, the measure of $\angle E = 104°$, and the measure of $\angle EBD = 110°$. What is the measure of $\angle D$?**



A 52°

INCORRECT: The student found $m\angle EBA$ but then thought $\angle EBA$ and $\angle DBC$ are congruent: $m\angle D \rightarrow 90° - 38°$.

B 55°

INCORRECT: The student thought $\angle EBA$ and $\angle DBC$ are congruent and used the fact that $\angle ABC$ is a straight angle:

$m\angle DBC \rightarrow \frac{180 - 110}{2} = 35°$ and so

$m\angle D \rightarrow 90° - 35°$.

C 58°

CORRECT: $\overline{AE} \cong \overline{BE}$ therefore $\angle EAB \cong \angle EBA$ because the base angles of an isosceles triangle are congruent. Since the sum of all interior angles of a triangle is 180°: $m\angle EBA = \frac{180° - 104°}{2} = 38°$.

Since $m\angle ABC$ is 180° (straight angle):

$m\angle DBC = 180° - 110° - 38° = 32°$.

Since acute angles of a right triangle are complementary: $m\angle D = 90° - 32° = 58°$.

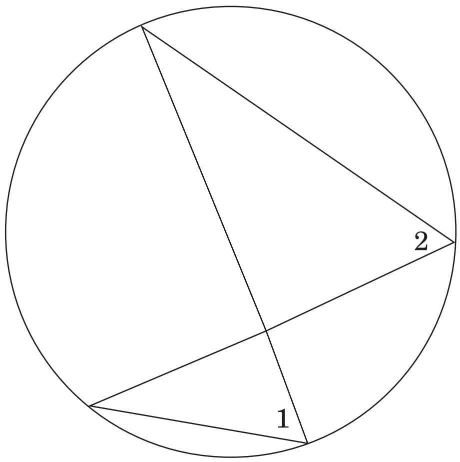
D 60°

INCORRECT: The student incorrectly assumed ΔBCD is a $30° - 60° - 90°$ triangle.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Congruence

8 **In the diagram, $\angle 1$ and $\angle 2$ are inscribed in the circle. How does the measure of $\angle 1$ compare to the measure of $\angle 2$?**



A The measure of $\angle 1$ is greater than the measure of $\angle 2$.

INCORRECT: The student thought since the triangle that includes $\angle 1$ is smaller than the triangle that includes $\angle 2$, the measure of $\angle 1$ must be greater than the measure of $\angle 2$ to share the same intercepted arc.

B The measure of $\angle 1$ is less than the measure of $\angle 2$.

INCORRECT: The student thought since the triangle that includes $\angle 1$ is smaller than the triangle that includes $\angle 2$, the measure of $\angle 1$ must be less than the measure of $\angle 2$.

C The measure of $\angle 1$ is equal to the measure of $\angle 2$.

CORRECT: Since $\angle 1$ and $\angle 2$ are inscribed angles and share the same intercepted arc, the angle measures are equal.

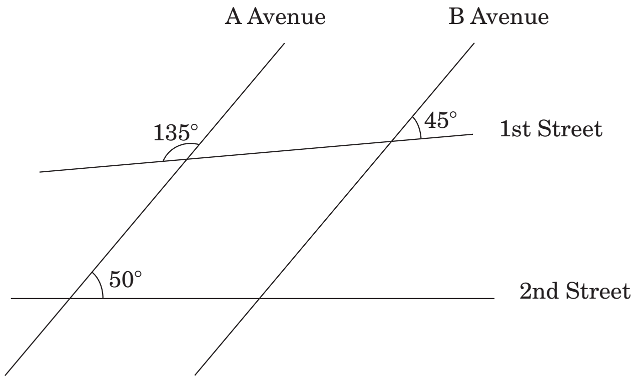
D Cannot be determined from the information provided

INCORRECT: The actual angle measures cannot be determined, however the measures are equal.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Circles

9 **While doing a research project about Vista City, Javier found the piece of an old city planner's map shown below. If all roads represented on the map are straight, what can Javier conclude about parallelism of the avenues and streets shown?**



A The avenues are parallel, but the streets are not.

CORRECT: Since the angles given formed by 1st Street with A Avenue and B Avenue are same-side exterior angles and are supplementary ($135° + 45° = 180°$), A Avenue and B Avenue are parallel. Since the avenues are parallel the measure of the acute angle formed by B Avenue and 2nd Street must be 50°. Therefore, 1st Street and 2nd Street are not parallel because the corresponding angles formed by B Avenue with 1st Street and 2nd Street are not congruent.

B The streets are parallel, but the avenues are not.

INCORRECT: The student reversed the roads that are parallel and not parallel.

C Both the avenues and the streets are parallel.

INCORRECT: The student did not apply the theorems on parallelism properly.

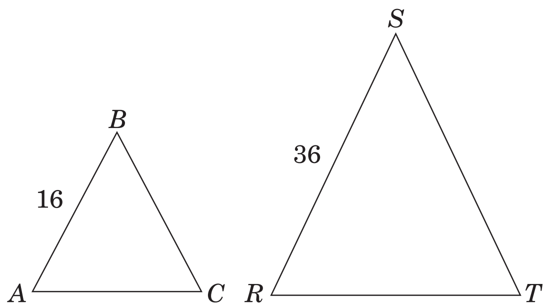
D Neither the avenues nor the streets are parallel.

INCORRECT: The student did not apply the theorems on parallelism properly.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Congruence

10 **$\triangle ABC$ is similar to $\triangle RST$. If $AB = 16$ and $RS = 36$, what is the ratio of the areas of the triangles?**



A 2 to 3

INCORRECT: The student found the square root of the ratio between corresponding sides.

B 4 to 9

INCORRECT: The student found the ratio between corresponding sides.

C 16 to 81

CORRECT: The ratio between the areas of two similar triangles can be found by taking the square of the ratio between corresponding sides. Since $\overline{AB}$ corresponds with $\overline{RS}$: $\left(\frac{16}{36}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

D Ratio cannot be determined from the information provided.

INCORRECT: The actual areas cannot be found and the student did not know how to use the ratio of corresponding sides to determine the ratio of the corresponding areas.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Similarity, Right Triangles, and Trigonometry

11 **Lindsey is making a rectangular sandbox. She measures one pair of opposite sides to be 15 feet and the other pair of opposite sides to be 12 feet. Lindsey concludes the sandbox doesn't look like a rectangle, but more like a parallelogram. What else could Lindsey verify to ensure the sandbox is a rectangle?**

A The diagonals are congruent.

> CORRECT: Rectangles are the only special parallelogram that have diagonals that are congruent.

B The opposite sides are parallel.

> INCORRECT: The student chose a characteristic that is true of all types of parallelograms.

C The diagonals bisect each other.

> INCORRECT: The student chose a characteristic that is true of all types of parallelograms.

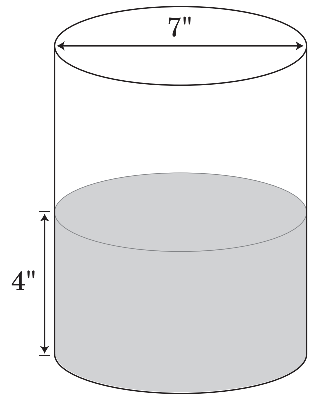
D The opposite angles are congruent.

> INCORRECT: The student chose a characteristic that is true of all types of parallelograms.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Congruence

12 **A paint can is in the shape of a cylinder with a diameter of 7 inches. The remaining paint in the can is 4 inches deep. To the nearest tenth, how many quarts of paint are remaining in the can? (1 quart = 57.75 in.^{3})**



A 1.5

INCORRECT: The student used an incorrect formula for the volume of a cylinder: $V \neq 2\pi rh$.

B 2.7

CORRECT: The volume of the paint remaining in the can is:

$$\begin{aligned} V &= \pi r^2 h \\ &\approx 3.14 \cdot (3.5)^2 \cdot 4 \\ &\approx 153.94 \text{ in.}^3 \end{aligned}$$

Using the conversion 1 quart = 57.75 in.^{3}, the number of quarts of paint in the can is:

$\frac{153.94}{57.75} \approx 2.7$ quarts

C 8.4

INCORRECT: The student used the diameter instead of the radius in the surface area formula ($S = 2\pi rh + 2\pi r^2$).

D 10.7

INCORRECT: The student used the diameter not the radius in the volume formula.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Geometric Measurement and Dimension

13 **Safety regulations require an extension ladder leaning against a wall to be angled so the distance between the bottom of the ladder and the wall is $\frac{1}{4}$ the length of the ladder. Assuming that the wall and the ground are perpendicular, how high (to the nearest foot) can a 24-ft. ladder be placed against a wall to meet the safety regulations?**

A 18 feet

> INCORRECT: The student found the difference between the length of the ladder and the distance between the bottom of the ladder and the wall.

B 23 feet

> CORRECT: The ladder, the ground, and the wall form a right triangle. Since the ladder is 24 feet long, the distance between the bottom of the ladder and the wall is 6 feet $\left(\frac{1}{4} \cdot 24\right)$. Let x equal the distance in feet the ladder is placed against the wall:
>
> $$\begin{aligned} 6^2 + x^2 &= 24^2 \\ 36 + x^2 &= 576 \\ x^2 &= 540 \\ x &\approx 23 \end{aligned}$$

C 25 feet

> INCORRECT: The student used the Pythagorean Theorem incorrectly: $6^2 + 24^2 = x^2$.

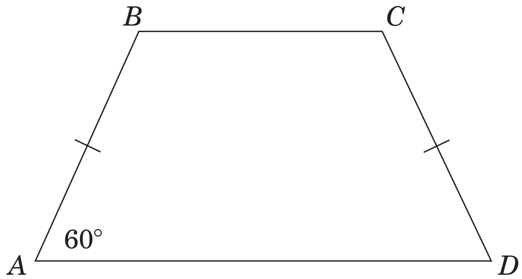
D 30 feet

> INCORRECT: The student found the sum of the length of the ladder and the distance between the bottom of the ladder and the wall.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Similarity, Right Triangles, and Trigonometry

14 ***ABCD* is an isosceles trapezoid. If the measure of $\angle A$ is 60°, what is the measure of $\angle C$?**



A 30°

> INCORRECT: The student thought $\angle C$ and $\angle D$ are complementary.

B 60°

> INCORRECT: The student thought the opposite angles in a trapezoid have the same measure.

C 120°

> CORRECT: Since the trapezoid is isosceles, the base angles are congruent, so $\angle A \cong \angle D$ ($m\angle D = 60°$). Since *ABCD* is a trapezoid, $\overline{BC} \parallel \overline{AD}$. Therefore, $\angle C$ and $\angle D$ are supplementary: $m\angle C = 180° - 60° = 120°$.

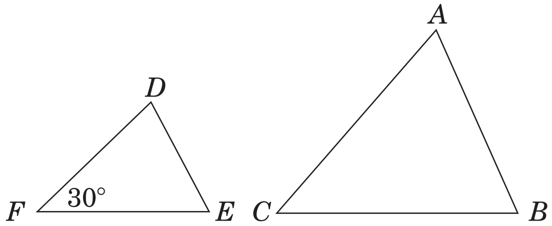
D Cannot be determined from the information provided

> INCORRECT: The student did not understand the properties of an isosceles trapezoid.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Congruence

15 **$\triangle DEF$ is the image of $\triangle ABC$ following a dilation with a scale factor of $\frac{1}{2}$. If the measure of $\angle F$ is 30°, what is the measure of $\angle C$?**



A 15°

INCORRECT: The student multiplied the angle measure by a scale factor of $\frac{1}{2}$.

B 30°

CORRECT: The corresponding angle measures in a dilation are equal.

C 60°

INCORRECT: The student multiplied the angle measure by a scale factor of 2.

D Cannot be determined from the information provided

INCORRECT: The student did not understand the properties of a dilation.

CCSS Conceptual Category:
Geometry

CCSS Domain:
Congruence